

# Flight Control and Scheduling for the Lateral Motion of a Tailless F-16 using Eigenstructure Assignment

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F-16

## Abstract

When one studies the history of aircraft design, one finds several attempts to design and build an aircraft with reduced tail size. Although this reduction has sometimes resulted in smaller drag and weight, most of these “tailless” aircraft had severe controllability problems, since the lateral-handling qualities were fairly poor. Regardless of these attempts, the interest in the development of aircraft with reduced tail size —compared with conventional aircraft — has recently increased. This interest is based on the intention of designing future aircraft by using more stealth technology in order to minimize their radar signature. One step to doing this is the reduction of the vertical tail which generates a large signature. The removal of parts of the vertical tail results in a lateral-directional behavior of the aircraft, which is far different than that of a conventional one. In order to get a new control on the lateral axis, a thrust vector control was added to the aircraft. It is obvious that without a reasonable controller the “tailless” aircraft cannot be handled by a pilot.

In this study, the method of the eigenstructure assignment will primarily be used to determine a controller for the “tailless” F-16. Using this method it is possible to design a simple closed-loop controller with a gain matrix in the feed-back loop. The eigenstructure assignment requires a desired system whose eigenstructure is taken to compute the gain matrix of the closed-loop system. A desired model will be developed which allows the placing of three of the four characteristic lateral motion eigenvalues of an aircraft — roll subsidence and two conjugate complex Dutch roll poles — on exact locations in the complex plane. At two different flight conditions, these parameters are varied to determine a controller which yields a advantageous behavior of the aircraft for a simulated roll rate and side slip. Following this, the desired model will be extended by actuator dynamics and

saturation. It will turn out that the parameters have to be chosen in a different way to prevent the system from becoming unstable. The simulation of the system will also reveal that there are some restrictions for the use of the developed controller. These refer primarily on limitations on the step size of the commanded impulse.

By using the results and the knowledge of the previously considered flight conditions — these include a low velocity and a high velocity operating point — a gain scheduling for intermediate velocities will be developed. This scheduling will be evaluated by simulating the response for the determined gain matrix for several velocities.

At the end of this study an LQ controller will be designed in order to see if this kind of controller is capable of yielding comparable results to the eigenstructure assignment. It will be observed that the closed-loop system responds in a different way to stabilize the aircraft after a commanded impulse. Furthermore, it will indicate that it is a problem to command a side slip and a roll rate as directly as for the previous system. One may attempt to improve this by introducing an asymptotic model following — without a basic improvement of the problem, as can already be inferred.

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## Table Of Variables And Acronyms

This table is meant to give an overview of the most important variables and acronyms used in the report. For multiply used variables and acronyms either description is specified. The respective meaning then results from the context in which the denotation is used.

### Variables

A	system matrix
B	control matrix
C	output matrix
D	direct feed-through matrix
g	universal constant of gravitation
h	altitude
H	transformation matrix — Hamilton matrix
i	imaginary operator
I	identity matrix
K	gain matrix
Ma	Mach number
p	roll rate
P	power — weighting matrix for the eigenstructure assignment
r	yaw rate
s	Laplace variable
u	control vector
v	eigenvector
V	velocity — modal transformation matrix
x	state vector
y	output vector
$\alpha$	angle of attack
$\beta$	side slip angle
$\delta$	actuator deflection
$\varepsilon$	actuator deflection
$\phi$	roll angle
$\gamma$	flight path angle
$\eta$	throttle setting
$\lambda$	eigenvalue
$\mu$	Lagrange multiplier
$\theta$	pitch angle
$\sigma$	real part of a complex value
$\tau$	break-off frequency
$\omega$	frequency — imaginary part of a complex value

$\psi$	yaw angle
$\zeta$	damping

### Subscripts - Indices

a	achievable/attainable
act	actuator
aero	aerodynamic
ail	aileron
cg	center of gravity
cl	closed-loop
cmd	commanded
d	desired
dr	Dutch roll
elv	elevator
eng	engine
ext	extended
ini	initial
max	maximum
out	output
spri	spiral
t	true
tv	thrust vector
0	trim point
1, 2, ...	numeration

### Superscripts

T	transposed
*	conjugate complex transposed